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Research Article

Error Analysis on the Concept of Composition of Function in Online Learning from the Perspective of AVAE Categories (ARITH, VAR, AE, and EQS)

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Abstract.
Algebra is known as a concept that is not only difficult to learn but also difficult to teach properly. Moreover, during the pandemic, learning was carried out online and many studies stated that online learning is not as effective as direct learning (face-to-face). The composition of function is one of the concepts in algebra where errors are often found. This study aims to analyze student errors on the concept of composition of functions in online learning from the perspective of AVAE categories (ARITH, VAR, AE, and EQS). To identify these errors, the researcher used a qualitative research method in which the data collection was done by giving written assignments to 25 high school students of grade X at one of the high schools in Sumedang. Some of the errors that were found in this study include errors in using commutative property, in addition operation (ARITH), students limitations in mastering the various roles of literal symbols (VAR), students not being able to decipher the order in algebraic expressions must be processed (AE), and students make mistakes because they ignore the form of equality in algebra (EQS). These errors that were found in this study can be used as a reference to identify more about learning obstacles that are felt by students which cause them to make errors.

Keywords: eror analysis, concept of composition, AVAE categories

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1. INTRODUCTION

Algebra is one of the most important fields in mathematics to learn. The basic ideas in algebra include: 1) algebra can be viewed as a generalization of arithmetic that includes counting strategies, 2) algebra as a mathematical language, including the meaning of variables, variable expressions, and meaning of completion, 3) algebra as a tool for studying mathematical functions and modeling, one of which includes activities to represent mathematical ideas using equations, tables, and graphs [1]. Algebra is the gateway to advanced mathematics and higher education [2].

Algebra is known as a concept that is not only difficult to learn but also difficult to teach properly [3]. This resulted in many difficulties felt by students when studying algebraic material [2, 4]. The weak algebraic abilities of Indonesian students can also be seen in the results of the 2011 TIMSS question analysis, which says that only 18% of Indonesian students answered correctly on the algebraic content of reasoning, moreover on application algebra content, only 1% of Indonesian students answered correctly [5].

The composition of function is one of the concepts delivered in class X of Senior High School (SMA) and is included in the scope of algebra. There are studies that say that there are still many students who make mistakes when studying composition of function concept, both students with high, medium, or low abilities [6]. Referring to the results of the study, there may be forms of errors that have not been identified by previous researchers, especially if the errors are categorized into the AVAEM category (ARITH, VAR, AE, EQS, and MATH). This AVAEM category is an error category that is specially arranged in the algebraic field [2]v. ARITH (Arithmetic) is an error in performing arithmetic operations related to operations, rules, and properties, VAR (Variable) is a student error related to understanding variables, AE (Algebraic Expression) is a student error in understanding algebraic expressions including parsing obstacle, expected answer obstacle, lack of closure obstacle, and lack of gestalt, EQS (Equal Sign) is a student error in understanding the different meaning of the "=" sign, and the last one is MATH (Mathematization) which is a student error in mathematizing [2]. It is just that in this study, the error category that the researcher used was only 4 categories of errors, namely ARITH, VAR, AE, and EQS (AVAE) because the composition function material discussed more symbolic mathematics, so the test questions given did not require students to do stages of mathematical modeling.

The determination of Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-COV-2), better known as the coronavirus, has become a global pandemic that has disrupted teaching and learning activities in schools [7]. This virus spreads very quickly so the learning process must be done online. Online learning is learning that is done without face-to-face directly but is done online. So that the obstacles felt by students between direct learning and indirect learning (online) will be different, which also results in significant differences in student learning outcomes [8]. Error analysis is a useful strategy to use in teaching and learning mathematics [9]. Therefore, it is important to conduct a study that analyzes in depth the concept of composition of function errors in online learning in terms of AVAE categories (ARITH, VAR, AE, and EQS).

2. METHOD

This study used a qualitative research method with a case study approach. By the advantages of the case study approach, the case study used in this study aims to analyze student errors in the composition function concept reviewed based on AVAE category. The participants in this study were 25 students from one of the senior high schools in Sumedang. The selection of participants in this study took into considers students who have heterogeneous abilities who have obtained material on the concept of composition function.

Due to the Covid-19 pandemic condition that is still happening in Indonesia, which causes mathematics learning in schools to be held online, the data collection process in this study was also collected online. Where students are asked to complete written assignments related to the concept of composition functions online using the help of Google Classroom. This written assignment consists of 5 subjective questions so that the form of the description question is chosen so that the researcher can see where the error is in solving the questions that students do.

3. RESULT AND DISCUSSION

Based on the results of collecting research data taken through written assignments on the concept of composition function involving 25 students of class X SMA, the researchers found several types of errors grouped into AVAE categories (ARITH, VAR, AE, and EQS) shown in Table 1.

TABLE 1: Number of students with avae error category.

Number of Questions	Number of Errors	AVAE Category			
		ARITH	VAR	AE	EQS
1a	3	2	-	1	-
1b	4	1	2	1	-
2a	4	3	1	-	-
2b	10	7	3	-	-
3a	13	12	1	-	-
3b	6	6	-	-	-
4a	4	2	1	-	1
4b	16	2	4	4	6
5	8	2	2	3	1

Based on Table 1, it can be seen that the most error findings were found in question number 4b, namely, 16 out of 25 students made mistakes, where students were asked to determine the formula for the function g if the formula for the composition function (gof) was known. When compared with the number of student errors when working on question number 4a, the findings of the error are not as much as in number 4b, question number 4a is the same as asking students to determine the function formula g if the composition function formula is known but the composition function formula that is known is the fog function formula so that the procedure for solving problem number 4a is different from the procedure for solving problem number 4b. The thing that causes the number of errors found in question number 4b is that there are still many students who consider the procedure for solving problem number 4a to be the same as solving problem number 4b. One study that is in line with this study also found that students still made many mistakes when determining the function formula g if the composition function formula was known, namely as many as 71.87% of students made mistakes [10]. If we look at the AVAE error category, the most common errors found are ARITH errors where this error is found in all questions. This happens because of the lack of student mastery of the prerequisite material. Findings in a similar study also said that the most common category finding was the ARITH category [11]v.

3.1. Arithmetic Category (ARITH)

Aritmetic category (ARITH) error is a form of error that expresses the limited mastery of students in 1) the use of symbolic expressions, in addition, subtraction, multiplication, and division, 2) the application of priority rules in arithmetic calculations, and 3) the use of the properties of numerical operations (commutative, associative, and distributive). Figure 1 presents some examples of findings in this study that fall into the ARITH category.

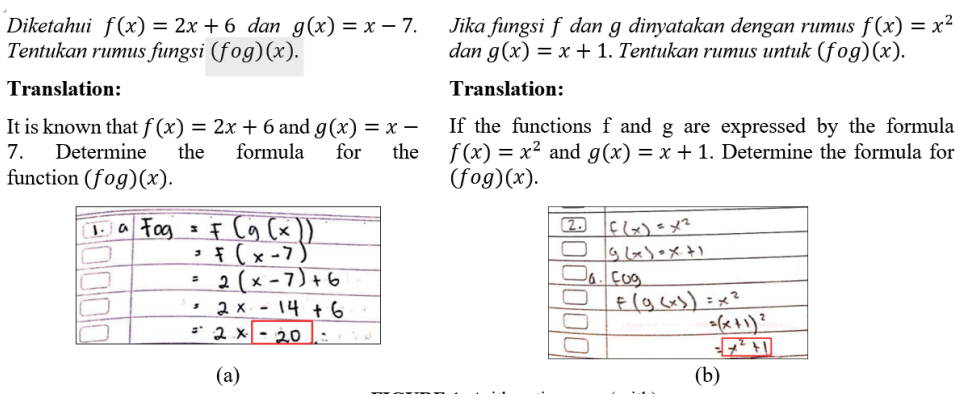


Figure 1: Arithmetic errors (arith).

Based on Figure 1 part a, it can be seen that students make mistakes in using the order rules when performing operations on numbers. Where the answer of -14+6 is -20. While in Figure 1 part b, students make mistakes in performing multiplication operations in algebraic form. Where students write (x+1)∧2=x∧2+1. Both forms of error belong to the ARITH category.

Diketahui fungsi $f(x) = 5x - 8$. *Tentukan rumus fungsi* $g(x)$ *jika* $(gof)(x) = 2 + 5x$.

Translation:
It is known that the function $f(x) = 5x - 8$. Find the formula for the function $g(x)$ if $(gof)(x) = 2 + 5x$.

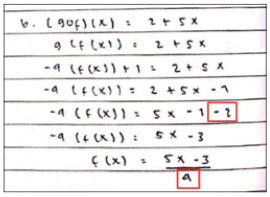


Figure 2: Arithmetic errors (ARITH).

Based on Figure 2, it can be seen that students make ARITH errors where students are wrong in using the commutative property in addition operations. Students assume that the form $-4(f(x)) = 2 + 5x - 1$ is equivalent to $-4(f(x)) = 5x - 1 - 2$, even though it should be in the form $-4(f(x)) = 2 + 5x\ -1$ is equivalent to $-4(f(x)) = 5x - 1 + 2$. The second error found when students changed the form $-4(f(x)) = 5x - 3$ to $f(x) = \frac{5x-3}{4}$, students were wrong in using the inverse property of multiplication. So all these errors are classified as ARITH errors.

3.2. Variable Category (VAR)

The campers subject experienced three difficulties in solving mathematical problems, namely difficulty in understanding the problem, difficulty in carrying out the plan, and difficulty in looking back. The first, the difficulty in understanding the problem, S-02 is able to identify the information that is known and asked for the questions, even though it is still incomplete. S-02 is also able to differentiate between square and rectangular concepts. However, S-02 made a mistake in making a mathematical model of the sentence, the width of Mr. Tatang's coconut garden is 15 m less than the length of the side of Mr. Sufyani's banana garden. S-02 is wrong by writing 15-x, when it should be x-15. The second is the difficulty in carrying out the plan, S-02 was wrong in solving an equation. S-02 performed the method of moving segments according to what her teacher taught her, but she forgot to change the sign so that the results obtained were

also wrong. The third is the difficulty in looking back, S-02 is confused about how to look back. S-02 was quickly satisfied with what was obtained.

3.3. Description of the difficulty of S-03 (quitters)

Variable category (VAR) errors that arise due to the limitations of students in mastering the various roles of literal symbols such as placeholder, generalized number, unknown, and varying quantities. Figure 2 presents some examples of findings in this study that fall into the VAR category.

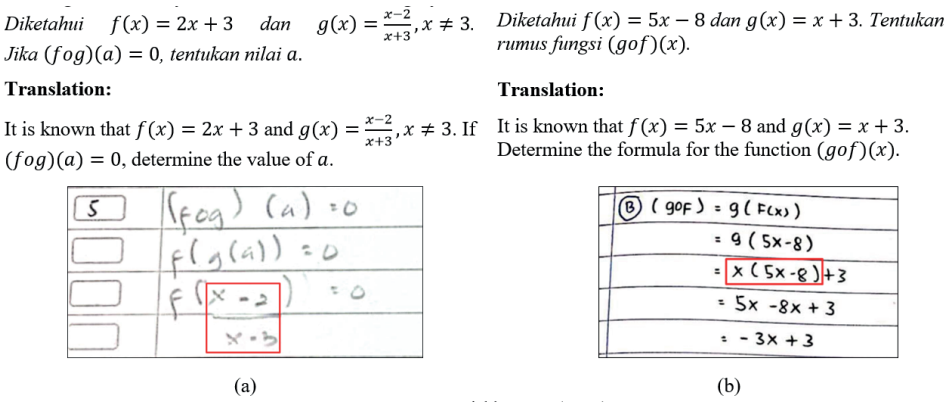


Figure 3: Variable errors (VAR).

Based on Figure 3 part a, it can be seen that the form of student error found is that students ignore the role of variables as varying quantities where the variable is used as a functional relationship either in terms of input arguments or as output function values. It can be seen that students write down the function rules $f(g(a))$ which is equivalent to $f(\frac{x-2}{x-3})$ even though the value of x should be changed to a so that the correct writing is $f(\frac{a-2}{a-3})$. Furthermore, in Figure 3 part b, students also ignore the role of variables as placeholders, where the input x in the function rule $f(x) = x + 3$ should be replaced with the input value $5x - 8$ to $(5x - 8) + 3$. The two types of errors fall into the category of VAR errors.

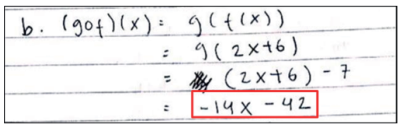
3.4. Algebraic Expression Category (AE)

Algebraic expression category (AE) is an error that arises because of the limitations of students about the operational meaning of algebraic expressions as a representation of the calculation process. Figure 3 presents some examples of findings in this study that fall into the AE category.

Diketahui $f(x) = 2x + 6$ *dan* $g(x) = x - 7$. *Tentukan rumus fungsi* $(fog)(x)$.

Translation:

It is known that $f(x) = 2x + 6$ and $g(x) = x - 7$. Determine the formula for the function $(fog)(x)$.

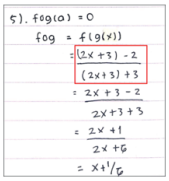


(a)

Diketahui $f(x) = 2x + 3$ *dan* $g(x) = \frac{x-2}{x+3}$, $x \neq 3$. *Jika* $(fog)(a) = 0$, *tentukan nilai* a.

Translation:

It is known that $f(x) = 2x + 3$ and $g(x) = \frac{x-2}{x+3}, x \neq 3$. If $(fog)(a) = 0$, determine the value of a.



(b)

Figure 4: Algebraic expression errors (AE).

Based on Figure 4 part a, it can be seen that students made an error in the AE category which was classified as a parsing obstacle because students were unable to decipher the order in which algebraic expressions had to be processed. Students use the distributive property to solve $(2x + 6) - 7$ even though between the terms $(2x + 6)$ and (-7) there is no multiplication operation so the correct solution should be $2x + 6 - 7 = 2x - 1$. While in Figure 4 part b, students make mistakes in the composition rules, where students are asked to find the fog composition function but what they use is gof calculations. Both forms of error belong to the category of AE (Algebraic Expression) error.

3.5. Equal Sign Category (EQS)

The EQS error category is an error that arises because of difficulties in understanding the difference in the meaning of the same sign as in arithmetic and algebra.

Diketahui fungsi $f(x) = 5x - 8$. *Tentukan rumus fungsi* $g(x)$ *jika* $(fog)(x) = 3 - 4x$.

Translation:
It is known that the function $f(x) = 5x - 8$. Find the formula for the function $g(x)$ if $(fog)(x) = 3 - 4x$.

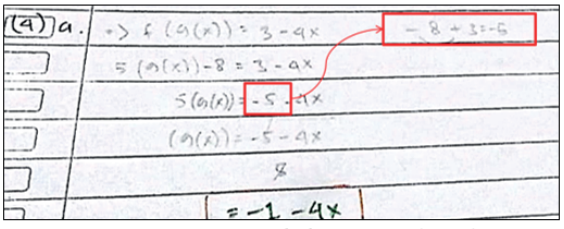


Figure 5: Equal sign errors (EQS).

Based on Figure 5, it can be seen that students make EQS errors where students ignore the form of equality in algebra. Students assume the form $5(g(x)) - 8 = 3 - 4x$ is equivalent to $5(g(x)) = -5 - 4x$. Even though the form $5(g(x)) - 8 = 3 - 4x$ should be equivalent to $5(g(x)) = 11 - 4x$. This error is classified as an EQS error.

4. CONCLUSION

Based on the results of research and data analysis, it can be concluded that there are still many student errors found in the concept of the composition function, including errors that are grouped into the AVAE category (ARITH, VAR, AE, and EQS) as follows: 1) ARITH category, namely the use of symbolic expressions, in addition, subtraction, multiplication, and division operations, and the use of the properties of numerical operations (commutative, associative, and distributive), 2) VAR category, namely students' limitations in mastering various roles of literal symbols, 3) AE category, namely students are not able to decipher the order in algebraic expressions that must be processed, and 4) EQS category, namely students make mistakes because they ignore the form of equality in algebra. The findings of these errors can be used as a reference to further identify the learning obstacles felt by students and then these learning obstacles can be studied again to create a better learning design.

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References

[1] Kriegler S. Just what is algebraic thinking? Introduction to Algebra Teaching; 2008. pp. 139–51.

[2] Jupri A, Drijvers P, van den Heuvel-Panhuizen M. Difficulties in initial algebra learning in Indonesia. Math Educ Res J. 2014;26(4):683–710.

[3] Watson A. Key understandings in mathematics learning, paper 6: algebraic reasoning. Oxford: University of Oxford; 2010.

[4] Herutomo AR, Saputro TE. "Analisis kesalahan dan miskonsepsi siswa kelas viii pada materi aljabar." Edusentris, Jurnal Ilmu Pendidikan dan Pengajaran. vol. 1, no. 2, pp. 134–145, 2014.

[5] Syamsul H, Novaliyosi N. “Timss Indonesia (trends in international mathematics and science study).,” In: Prosiding Seminar Nasional & Call For Papers. pp. 562–569 (2019).

[6] Kolins AY, Wahyuningsih W, Safrudin N, Rusdin ME. Analisis kesalahan peserta didik dalam menyelesaikan soal matematika pada fungsi komposisi dan fungsi invers. AlphaMath : Journal of Mathematics Education. 2020;6(2):86.

[7] WHO, IFRC, and UNICEF, Key messages and actions for prevention and control in schools., 2020.

[8] Lassoued Z, Alhendawi M, Bashitialshaaer R. An exploratory study of the obstacles for achieving quality in distance learning during the covid-19 pandemic. Educ Sci (Basel). 2020;10(9):1–13.

[9] Rushton SJ. Teaching and learning mathematics through error analysis. Fields Mathematics Education Journal. 2018;3(1):1–12.

[10] Suharto S, Widada W, Susanta A, Haji S. The ability to understand concepts: cognitive style, cognitive structure, learning styles and learning motivation. PENDIPA Journal of Science Education. 2020;5(1):15–22.

[11] Zulfa BI, Suryadi D, Fatimah S, Jupri A. Student’s mistake in algebraic fraction: an analysis using avae categories. J Phys Conf Ser. 2020;1521(3):1–6.