Research Article

Mathematics Learning Assessment Based on Pirie-Kieren's Theoretical Framework in Elementary School

Ramadhan Kurnia Habibie Negara[1], Turmudi Turmudi[2], and Wahyudin Wahyudin[2]

[1]Departement of Mathematics Education, Universitas Islam Negeri Mataram, Indonesia
[2]Departement of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia

ORCID
Ramadhan Kurnia Habibie Negara: https://orcid.org/0000-0002-2815-7427
Turmudi Turmudi: https://orcid.org/0000-0001-7976-211X
Wahyudin Wahyudin: https://orcid.org/0000-0003-3930-1380

Abstract.
Various non-technical factors in the mathematics learning assessment process carried out during the COVID-19 pandemic made it difficult to describe students' mathematical understanding as a whole. This causes a lot of gaps in students' mathematical understanding. This study aims to describe the growth of elementary school students' understanding of geometry through the use of an assessment instrument based on the Pirie-Kieren theoretical framework. This study uses a qualitative approach with a case study method. The participants in this study were fifth-grade elementary school students who were studying the topic of geometry. Data on the growth of students' understanding of geometry was obtained through the process of analyzing the results of students' work in completing the geometry understanding growth test based on the Pirie-Kieren theoretical framework. Findings from student work were then followed up with in-depth interviews. The results of interpretational constructivism data analysis through data reduction, data presentation, and conclusions show that there are gaps in students' understanding of geometry topics. The gap can be observed from every inhibitor and folding back activity in each layer of student understanding.

Keywords: elementary school, mathematics learning assessment, pirie-kieren's theoretical framework.

Corresponding Author: Ramadhan Kurnia Habibie Negara; email: habibiperwira@uinmataram.ac.id

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1. INTRODUCTION

Nowadays, the growth of education in Indonesia is going dynamic in various aspects, including the learning process. Unfortunately, this happened without being followed by changes in the principles of learning assessment. So far, the learning assessment developed in elementary schools has only been limited to the assessment of learning outcomes. The test instrument used only measures cognitive abilities and produced learning outcomes scores that cannot be used to measure students' abilities comprehensively. The existing learning outcomes assessment system can change the nature of learning from increasing student potential on an ongoing basis to an intensive activity of working on practice questions for specific exams [1, 2]. The current learning assessment practice is no longer following the Regulation of the Minister of Education and Culture of the Republic of Indonesia Number 43 of 2019 concerning the Implementation of Examinations Organized by Education Units and National Examinations. The regulation is a written instruction from the state to all practitioners in the classroom to change the paradigm of learning assessment that they have been doing so far. The learning process must return to its initial function, namely increasing student potential optimally and sustainably [3].

The Covid-19 pandemic has brought various sporadic changes in educational practice [4–6]. All kinds of learning activities that previously could be carried out face-to-face can no longer be carried out and are replaced with online modes. Students and teachers are forced to adapt to the existing conditions. Some schools with qualified human resources and conditions can adapt by carrying out learning assisted by the application zoom meeting, google classroom, jitsimeet, umeetme, and other applications. While the rest only rely on WhatsApp message-assisted learning with various limitations. This is in line with the implementation of learning assessments during the Covid-19 pandemic. Several schools with qualified human resources and conditions can develop applications that support the online learning assessment process. While the rest only rely on sending exam questions via WhatsApp chat where later answers will also be sent from the same mode without any further supervision. This inequality is further strengthened by the unequal distribution of facilities and quality of education in each education unit [7, 8]. Starting from the unequal socio-anthropological conditions of each student and their environment to providing tools to participate in online learning modes. These various technical and non-technical factors resulted in the assessment of mathematics learning carried out during the Covid-19 pandemic being unable to describe students' understanding of mathematics as a whole.

Students at the elementary school level are generally in the range of 7-12 years, where based on Piaget's theory of development (1976) students are in the concrete operational stage so that student understanding is built by inductive reasoning processes [9, 10]. Understanding is obtained through the adaptation of students' mental structures that depend on information obtained through their experiences [11]. The new information obtained is incorporated into existing mental structures and then adapted to process new information and knowledge [12]. Understanding involves a dynamic mental process which there is a gradual process that includes the activities of translating, interpreting, exploring, analyzing, synthesizing, and evaluating. Mathematics is included in the hierarchical science category where the concepts and principles presented earlier are the basis or reference for developing further concepts. Developing this understanding makes mathematics an effective thinking tool because it uses consistent reasoning rules.

Mathematical thinking is not exclusive thinking in mathematics but can be generalized and used in various problems [13, 14]. Mathematics is not only present as a material content that must be understood but also as a way or pattern of thinking. The study of mathematics is abstract, where the objects studied include facts, operations, concepts, and principles [15, 16]. The opposite characteristics between the object of mathematical study and the cognitive development stages of elementary school students are the cause of the emergence of these difficulties. Geometry as one of the mathematics topics studied in elementary school has a role as an initial bridge to introduce mathematics as something abstract through concrete experiences [17, 18]. Geometry presents abstractly a person's visual and spatial experience in their mental structure which can train students to be able to reason mathematically to increase their confidence and mathematical communication skills [19, 20]. As a hierarchical and recursive science, understanding mathematical concepts is highly dependent and related to other concepts in mathematics itself [21, 22]. Therefore, it is very natural to give the best attention and focus to all mathematics learning topics in general.

The theory of Pirie & Kieren (1994) has provided a theoretical framework for the growth of mathematical understanding, divided into eight levels (layers) of understanding. The framework consists of layers of primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventing. The growth of mathematical understanding is a dynamic, active, continuous, but not linear, and recursive process that involves the learning environment and the response of the learning ecosystem itself [23–25]. Thus, to gain understanding, students must be active in building newly acquired knowledge with relevant knowledge that students have previously had [11, 26]. The Pirie-Kieren model provides a framework for mapping student actions in various contexts, tracing dynamic movements across eight levels of understanding. The search is carried out in the stages of students building, searching, and collecting ideas [27].

Assessment is an integral part of the learning process. Assessment is the process of planning, obtaining, and providing information for teachers to make alternative decisions in the ongoing and future learning process [28]. The role of assessment is not only part of learning, but the assessment has a vital function for learning itself. Written tests cannot be used to measure students' abilities comprehensively. However, these tests are still helpful in measuring students' mastery of the knowledge base. Therefore, learning that entirely refers to the test will not positively value student progress.

In contrast with past assessments that only focused on learning outcomes, the principle of assessment for learning can measure student abilities comprehensively, essential skills, processes, and student investigations [29, 30]. Assessment for learning is based on the understanding that the learning process can and should occur through the assessment process. Assessment for learning is a form of assessment carried out to improve learning, not just a tool to see the amount of knowledge possessed by students [31–33]. Students are positioned as partners in the learning process who have an interest in and responsibility for their learning. Students are expected to be able to evaluate themselves (self-assessment) so that they will better understand the problems (problems) they have faced [34, 35]. The teacher has the task of appreciating the student's capacity to support the improvement of students' abilities in conducting evaluative assessments [36]. The application of assessment for learning in learning mathematics can increase motivation, self-confidence, self-awareness, student behavior during learning, and ability in mathematics [37].

This study explicitly discusses the growth of students' mathematical understanding when they learn geometry which is assessed with an assessment instrument developed based on the Pirie-Kieren theoretical framework. As has been explained that elementary school students are in the concrete operational stage, and geometry is chosen to introduce mathematics as something abstract through concrete experience. The assessment instrument has been developed following the principle of assessment for learning by considering the eight understanding layers of Pirie-Kieren's theory. The picture of understanding growth resulting from the assessment process can be used as an indicator of the achievement of students' mathematical understanding of geometry in detail.

The gaps in students' mathematical understanding that occurred during Covid-19 pandemic learning will be analyzed in-depth with the help of an assessment instrument based on the Pirie-Kieren theory. This study will explore gaps in the description of students' understanding at each level of understanding. Each acting and expressing activity is described in detail, accompanied by the findings of inhibitors and folding back activities at each layer of understanding. These findings were collected and then used as complete information about the gaps in students' mathematical understanding that could be reflected in mathematics learning.

2. RESEARCH METHOD

This study uses a qualitative approach with a case study method. This approach describes the growth of students' mathematical understanding to find gaps in students' mathematical understanding with an in-depth and detailed investigation process. The participants in this study were fifth-grade elementary school students studying the topic of geometry that represent students with moderate mathematical abilities. The researcher observed the learning process and focused on observing the participants during the task-completion process. Data on the growth of students' understanding of geometry was obtained by analyzing the results of students' work in completing the geometry understanding growth test based on the Pirie-Kieren theoretical framework. After completing the test, the student then followed up with in-depth interviews. The data findings are then interpreted and analyzed in a constructivist manner through data reduction, data presentation, and drawing conclusions, indicating that there are gaps in students' understanding of the topic of geometry. This gap can be seen from the description of students' mathematical understanding at each layer, including acting, expressing, and folding back activities. This description will appear inhibitors that can be used as a reference in determining the gaps in students' mathematical understanding. The growth line of students' mathematical understanding will be depicted in the form of a mathematical understanding diagram that refers to the Pirie-Kieren theoretical framework.

3. RESULTS AND DISCUSSION

Following the theory proposed by Pirie & Kieren (1994), the growth in understanding of the two participants is dynamic and continuous, but not linear. The study found that the two participants reached different layers of understanding. In addition, the folding back activities carried out also have different characteristics. All understanding activities come from the participants themselves without any trigger from the researcher.

3.1. Growth of Mathematical Understanding of First Subject

From the research data, it is known that the first subject only reaches the layer of property noticing on the quadrilateral area topic (Figure 1). Subjects set out to understand the quadrilateral area (parallelogram, trapezoid, and kite) by bringing primitive knowing in the form of previous knowledge about the rectangle area. In the image-making layer, the subject's acting (image doing) appears from the subject's ability to use prior knowledge of area concepts to use tangram kits (A). While the subject's expressing (image reviewing) looks from the subject's ability to articulate his understanding of the area of quadrilateral obtained from the activity of media kit tangram usage.

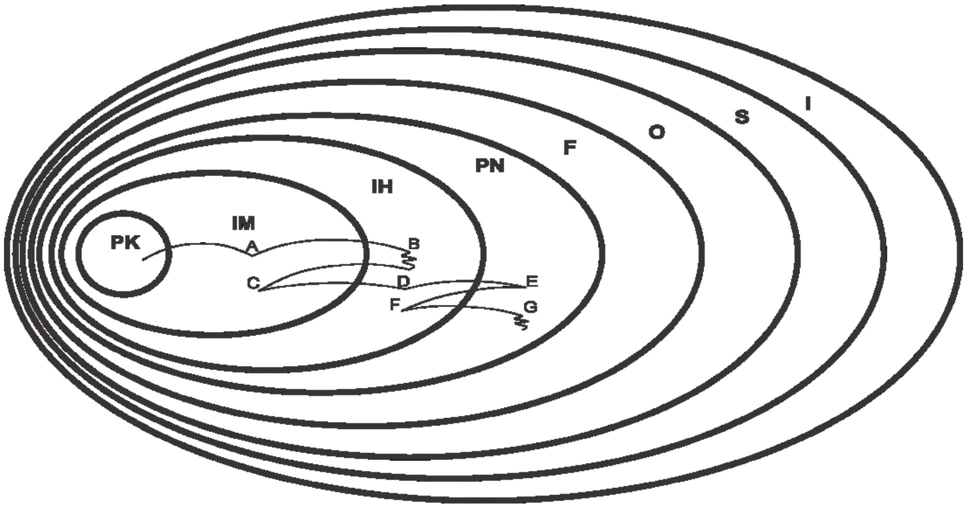

Figure 1: Growth of mathematical understanding of first subject.

In the image having layer, the understanding of the subject is a mental picture of the quadrilateral area. Acting's subject (image seeing) in the form of action in finding a formula for calculating the area of a quadrilateral with a comprehensive look at its association with rectangle area (B). Shaping the understanding of the subject requires folding back into the image making layer. The subject works to a deeper layer without leaving the topic to recall his current understanding [38]. Folding back comes with the subject trying to recall the images made with the kit tangram media (C). The subject recalls the puzzle linkage used in the tangram media kit to determine the formula to be used to find the area of the quadrilateral (D). The subject's expressing (image saying) in the form of the ability to articulate the reason for obtaining the formula to find the quadrilateral area. The subject requires a trigger from the researcher to reveal the folding back activity [39] performed and the reason for getting the formula to find the quadrilateral area. The subject sees the linkage of the puzzle arrangement used to determine the formula to be used to find the area of the quadrilateral.

In the layer of property noticing, the subject's acting (property predicting) is in the form of the ability to pay attention to the properties of a quadrilateral to calculate the area of the quadrilateral (E). This study found that understanding of the subject at this layer is not yet complete. The activities are undertaken, and the interview found that the subject only understands the properties and formulas to find the extent of the parallelogram. While understanding the properties and formulas to find the area of a trapezoid or kite, the subject still needs a trigger to do folding back activity into the image having layer. The subject tries to work on a deeper layer to recall the formula for finding the trapezoidal area and the kite and the reason for obtaining that formula (F). But folding back activity of the subject this time is not effective in expanding his understanding [40]. It inhibits the subject's ride to the next layer of understanding (formalizing) (G). Consequently, expressing the subject (property recording) cannot explain all the quadrilateral fields correctly. The subject is confused because his understanding of the previous layer has not matured. So it becomes the subject's inhibitor of this layer to move on to the next layer of understanding.

3.2. Growth of Mathematical Understanding of Second Subject

From the data of the research results, it is known that the second subject only reaches the formalizing layer on the quadrilateral area topic (Figure 2). Subjects set out to understand the quadrilateral area (parallelogram, trapezoid, and kite) by bringing primitive knowing in the form of previous knowledge about the rectangle area and area conservation. In the image-making layer, the subject's acting (image doing) appears from the subject's ability to use prior knowledge of area concepts and area conservation concepts to use tangram kits (A). While the subject's expressing (image reviewing) looks from the subject's ability to articulate his understanding of the area of quadrilateral obtained from the activity of media kit tangram usage.

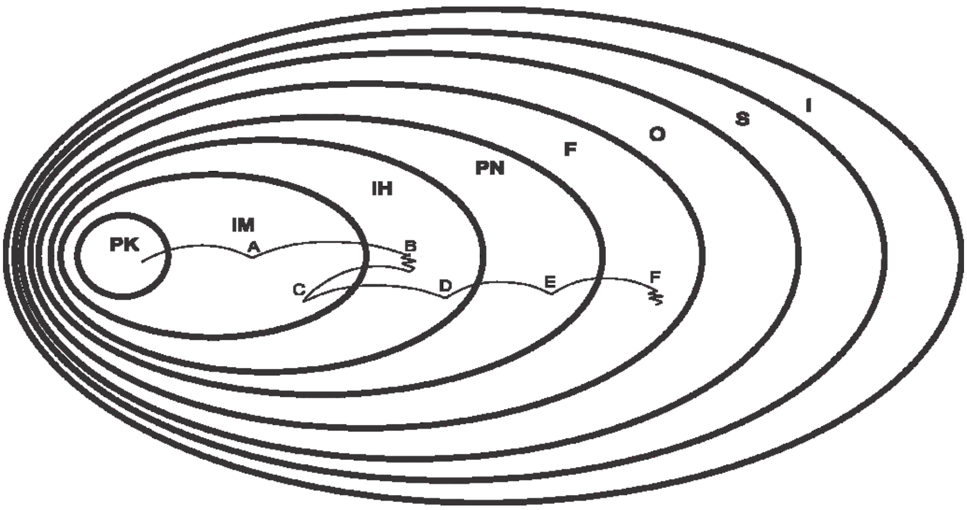

Figure 2: Growth of mathematical understanding of second subject.

In the image having layer, the understanding of the subject is a mental picture of the quadrilateral area. Acting's subject (image seeing) in the form of action in finding a formula for calculating the area of a quadrilateral with a comprehensive look at its association with rectangle area (B). In this layer, the subject does a little folding back to work into the inner layer (C). The subject's folding back into the image making layer is to help find information to broaden his understanding (D) [41]. The subject's expressing (image saying) in the form of the ability to articulate the reason for obtaining the formula to find the area of the quadrilateral.

In the layer of property noticing, the subject's acting (property predicting) is in the form of the ability to pay attention to the properties of a quadrilateral to calculate the area of the quadrilateral (E). The subject's expressing (property recording) is the ability to articulate the subject's understanding of quadrilateral properties and formulas to calculate the quadrilateral area. In the formalizing layer, the subject acting (method applying) is the ability to see the characteristics of the quadrilateral to use it in searching the area of the quadrilateral. But unfortunately, in this layer of understanding, the subject is inhibited from applying algorithmic activity due to the subject's ability in the properties of arithmetic operations being less well. So the subject is inhibited from moving up to the next layer of understanding (F) [42]. Consequently, the subject's expressing (method justifying) cannot articulate the algorithmic activity undertaken. So the subject can not reach the next layer of understanding.

4. CONCLUSION

The growth of students' mathematical understanding took place dynamically, and continuously, but not linearly. The gaps in mathematical understanding found in the two subjects are quite different. The first subject only reached a layer of understanding property noticing by experiencing two folding back activities. The inhibitor found the subject's weakness in reconstructing geometric shapes in his mental image. While on the second subject, the understanding layer achieved is formalizing. The second subject only did one folding back activity to strengthen his understanding by reconstructing the information obtained in the previous layer. Unfortunately, the second subject cannot grow to the next layer because he gets an inhibitor in the form of an inability to master the arithmetic prerequisite material in that layer. The finding of gaps in the description of understanding can be used as important information to reflect on learning activities. So that the holes created during the learning process during the Covid-19 pandemic can be closed immediately. Future research is expected to cover a more comprehensive range of topics and participants for a better mapping process for the growth of mathematical understanding.

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References

[1] V. Hatisaru, “School students depictions of mathematics teaching and learning practices.,” *International Electronic Journal of Elementary Education*. vol. 13, no. 2, pp. 199–214, 2021. https://doi.org/10.26822/iejee.2021.184.

[2] Kinnear G, Wood AK, Gratwick R. Designing and evaluating an online course to support transition to university mathematics. Int J Math Educ Sci Technol. 2022;53(1):11–34.

[3] Bird FL, Yucel R. Improving marking reliability of scientific writing with the developing understanding of assessment for learning programme. Assess Eval High Educ. 2013;38(5):536–53.

[4] Nguyen T, Netto CL, Wilkins JF, et al. “Insights into students’ experiences and perceptions of remote learning methods: From the covid-19 pandemic to best practice for the future,” *Frontiers in Education*. vol. 6, p. 2021.

[5] Zhang C, Yan X, Wang J. EFL teachers’ online assessment practices during the covid-19 pandemic: changes and mediating factors. Asia-Pac Educ Res. 2021;30(6):499–507.

[6] Kalloo RC, Mitchell B, Kamalodeen VJ. Responding to the Covid-19 pandemic in Trinidad and Tobago: challenges and opportunities for teacher education. J Educ Teach. 2020;46(4):452–62.

[7] Kawuryan SP, Sayuti SA, Aman A, Dwiningrum SI. Teachers quality and educational equality achievements in Indonesia. Int J Instr. 2021;14(2):811–30.

[8] B. Kharisma and V. Pirmana, “The role of government on education quality and its provision: The case of public junior secondary school among provinces in Indonesia.,” *European Journal of Social Sciences*. vol. 37, no. 2, pp. 259–70,.

[9] Florea NM, Hurjui E. Critical thinking in elementary school children. Procedia Soc Behav Sci. 2015;180(5):565–72.

[10] Perret P. Children’s inductive reasoning: developmental and educational perspectives. J Cogn Educ Psychol. 2015;14(3):389–408.

[11] Bakar MA, Ismail N. Mathematical instructional: A conceptual of redesign of active learning with metacognitive regulation strategy. Int J Instr. 2020;13(3):633–48.

[12] B. Sel, “The effect of conceptual change texts on the level of conceptual understanding of students.,” *International Electronic Journal of Elementary Education*. vol. 11, no. 4, pp. 383–391, 2019. https://doi.org/10.26822/iejee.2019450796.

[13] Attridge N, Inglis M. Increasing cognitive inhibition with a difficult prior task: implications for mathematical thinking. ZDM Math Educ. 2015;47(5):723–34.

[14] Calao LA, Moreno-León J, Correa HE, Robles G. Developing mathematical thinking with scratch. Lect. Notes Comput. Sci. 2015. pp. 17–27.

[15] Ernest P. “Mathematics and Values.,” In: *Trends in the history of science [Internet*. pp. 189–214., Birkhäuser, Cham (2016). https://doi.org/10.1007/978-3-319-28582-5_12.

[16] Marquis JP. Categorical foundations of mathematics or how to provide foundations for abstract mathematics. Rev Symb Log. 2013;6(1):51–75.

[17] Jelatu S, Sariyasa S, Ardana IM. Effect of GeoGebra-Aided REACT strategy on understanding of geometry concepts. Int J Instr. 2018;11(4):325–36.

[18] Sarama J, Clements DH. “Physical and virtual manipulatives: What is ‘concrete’?” In: *Mathematics Education in the Digital Era [Internet]. Int*. pp. 71–93. *Springer*, Cham (2016). https://doi.org/10.1007/978-3-319-32718-1_4.

[19] Diana N, Suryadi D, And JA. Dahlan, “Analysis of students’ mathematical connection abilities in solving problem of circle material: transposition study,”. Journal for the Education of Gifted Young Scientists. 2020;8(2):829–42.

[20] Ramlan AM. The effect of van hiele learning model toward geometric reasoning ability based on self-efficacy of senior high school students [Journal of Mathematics Education]. JME. 2016;1(2):62–71.

[21] Cvencek D, Kapur M, Meltzoff AN. Math achievement, stereotypes, and math self-concepts among elementary-school students in Singapore. Learn Instr. 2015;39:1–10.

[22] Ramirez G, Shaw ST, Maloney EA. Math anxiety: past research, promising interventions, and a new interpretation framework. Educ Psychol. 2018;53(3):145–64.

[23] Pirie S, Kieren T. Growth in mathematical understanding: how can we characterise it and how can we represent it? Educ Stud Math. 1994;26(2–3):165–90.

[24] Sengul S, Yildiz SG. An examination of the domain of multivariable functions using the pirie-kieren model. Universal Journal of Educational Research. 2016;4(7):1533–44.

[25] Wright V. Frequencies as proportions: using a teaching model based on Pirie and Kieren’s model of mathematical understanding. Math Educ Res J. 2014;26(1):101–28.

[26] Abrahamson D, Trninic D. Bringing forth mathematical concepts: signifying sensorimotor enactment in fields of promoted action. ZDM Math Educ. 2015;47(2):295–306.

[27] Gokalp ND, Bulut S. A new form of understanding Maps: multiple representations with pirie and kieren model of understanding. International Journal of Innovation in Science and Mathematics Education. 2018;26(6):1–21.

[28] Sawant DG. "Role of IQAC in maintaining quality standards in teaching, learning and evaluation.," *Pacific Science Review B*. Humanities and Social Sciences. 2016;2(2):66–9.

[29] Heitink MC, Van der Kleij FM, Veldkamp BP, Schildkamp K, Kippers WB. A systematic review of prerequisites for implementing assessment for learning in classroom practice. Educ Res Rev. 2016;17:50–62.

[30] Smaill E. Using involvement in moderation to strengthen teachers' assessment for learning capability. Asess Educ. 2020;27(5):522–43.

[31] Laveault D, Allal L. "Implementing assessment for learning: Theoretical and practical issues.," Presented at the (2016). https://doi.org/10.1007/978-3-319-39211-0_1.

[32] Chng LS, Lund J. Assessment for learning in physical education: the what, why and how. J Phys Educ Recreat Dance. 2018;89(8):29–34.

[33] Shute VJ, Rahimi S. Review of computer-based assessment for learning in elementary and secondary education. J Comput Assist Learn. 2017;33(1):1–19.

[34] Barana A, Marchisio M, Sacchet M. "Advantages of using automatic formative assessment for learning mathematics.," Presented at the (2019). https://doi.org/10.1007/978-3-030-25264-9_12.

[35] Hawe E, Dixon H. Assessment for learning: a catalyst for student self-regulation. Assess Eval High Educ. 2017;42(8):1181–92.

[36] Thomson R. Implementation of criteria and standards-based assessment: an analysis of first-year learning guides. High Educ Res Dev. 2013;32(2):272–86.

[37] Winarno W, Zuhri M, Mansur M, Sutomo I, Widhyahrini K. Development of assessment for the learning of the humanistic model to improve evaluation of elementary school mathematics. Int J Instr. 2019;12(4):49–64.

[38] V. Martin, *Leading Change in Health and Social Care*. Routladge, London. https://doi.org/10.4324/9780203714157.

[39] Yao X, Manouchehri A. Folding back in students' construction of mathematical generalizations within a dynamic geometry environment. Math Educ Res J. 2022;34(2):241–68.

[40] Gulkilik H, Moyer-Packenham PS, Ugurlu HH, Yuruk N. Characterizing the growth of one student's mathematical understanding in a multi-representational learning environment. J Math Behav. 2020;58:100756.

[41] Luo L. “Recursion in the mathematics curriculum.,” *Philosophy of Mathematics Education Journal*. vol. 28, p. 2014.

[42] Towers J, Martin LC. Building mathematical understanding through collective property noticing. Canadian Journal of Science, Mathematics and Technology Education. 2014;14(1):58–75.