KnE Social Sciences
ICMScE
International Conference On Mathematics And Science Education
Volume 2024

Research Article

Students' Errors in Constructing Mathematical Proofs by Direct Method

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Published: 26 April 2024

Publishing services provided by Knowledge E

Selection and Peer-review under the responsibility of the ICMScE Conference Committee.

Abstract.
In mathematics, there are several methods for proving mathematical statements, two of them are direct method and indirect method. Both the methods were included in the mathematics curriculum at senior high school level as a learning topic. It implied that the teachers or pre-service teachers must master these methods. In addition, learning advanced mathematics courses also needed the ability of mathematical proving. The direct method was often used in proving mathematical statements. This descriptive qualitative study was conducted to examine what errors were written by the students in constructing mathematical proofs by direct method. Two basic problems (mathematical statements) were administered to 13 college students. They were pre-service mathematics teachers in one of the public universities in Aceh, Indonesia, and were asked to prove the statements by direct method. The data were analyzed by using Miles and Huberman step. The results showed that there were three main errors, (1) errors in using definition correctly, (2) errors in algebraic process, and (3) proving by examples. It concluded that the students must be accustomed to mathematical proving activities to avoid errors in constructing mathematical proofs.

Keywords: constructing mathematical proofs, direct method, students' errors.

1. INTRODUCTION

Mathematical proving is one of the abilities that must be improved during teaching and learning process in secondary school [1]. National Council of Teachers of Mathematics (NCTM) also included proof as one of standard process in learning mathematics jointly with reasoning [2]. In secondary school, there are many formulas and statements that must be derived/verified through mathematical proving activities. For instances, the formula of volume and surface area of three-dimensional objects in junior high school must been proved as well as materials in senior high school such as permutation and combination formula also must be proved in teaching and learning process. Moreover, in Senior High School, there is one topic specific about mathematical proving. It is techniques of mathematical proving. It indicates that mathematical proving ability to construct mathematical proofs is essential.

Mathematical proof is useful for the students. One of the aims of mathematical proof is to develop students' critical thinking skill and improve mathematical high order thinking and reasoning [3, 4]. Mathematical proof is also as a tool for improving students to comprehend mathematical concepts [5]. Due to the importance of mathematical proof for the students, the teachers have to involve mathematical proving activities in the mathematics classroom. The mathematical proving activities divided into two i.e., comprehending mathematical proof and constructing mathematical proof. For comprehending mathematical proof, a study showed that the students ability in proof comprehension still needed to be improved [6]. For constructing mathematical proof, there were many studies shown the students difficult in it [7–10]

In mathematics, many techniques can be used to construct mathematical proof. They are direct methods, proof by cases, contradiction, induction, and contrapositive methods [11]. In other references, contradiction and contrapositive are included in indirect methods. In Indonesia Senior High School's curriculum, all of the techniques except proof by cases were introduced in the topic of mathematical logic. It implies that constructing formal mathematical proofs is one of the skills that must be mastered by the students. Since mathematical proof was the topic in senior high schools, it was imperative to make sure that college students in mathematics education study program as pre-service teachers have to expert in it. As an effort to accomplish this goal, a study was conducted to identify college students' error in constructing mathematical proof by direct method. The aim of the study was to identify what errors made by the students so that in the teaching and learning process in the college, lecturers and students would aware about the errors when proving mathematical statements.

2. RESEARCH METHOD

The focus of this study was to explain students' errors in constructing mathematical proofs by direct method deeply. Therefore, the method of this study is qualitative method by descriptive type. It was in line with the definition of qualitative method which is a method designed to provide deep explanation of a specific program or setting [12].

The participants of the study were thirteen students of mathematics education study program in one of public universities in North Aceh, Aceh, Indonesia. There were two reasons of selecting the participants, which were (1) they had learned about mathematical proving methods and (2) they have learned several advanced mathematics courses that included mathematical proving ability. To identify the errors in constructing mathematical proof by direct methods, two problems were administered to them. The questions were the common mathematical statements. The Questions were:

1. Prove that if x is even, then $x^2 - 8x + 3$ is odd number.
2. Suppose that a is the product of four consecutive integers. Prove that $a + 1$ is perfect square number.

The students were asked to prove the statements by using direct method type of mathematical proofs. The answers were analysed for each question using Miles and Huberman step, i.e. data reduction, data display, and drawing conclusion [13].

3. RESULTS AND DISCUSSION

The purpose of this study was to identify what errors were made by the students in constructing mathematical proofs by direct method. To accomplish the purpose, two questions were given to them. Here are the errors made by the students in proving mathematical statements in each question.

3.1. The Students' Errors in Question Number 1

In question number 1, the students were asked to prove simple mathematical statements. The question was “Prove that if x is even, then $x^2 - 8x + 3$ is odd number”. To prove this statement, the students have to know several concepts. The first is the general form of even numbers (definition of even number). They must know that the form of even numbers is $2k$, k is element of integers (suppose that $x = 2k$). Then, substitute each x by $2k$ to the form $x^2 - 8x + 3$. In this step, the ability of basic algebraic manipulation was needed. If the student expert in the ability, they will success in this problem.

The result in question number 1 showed that all of the students knew what hypothesis or assumption and conclusion of a mathematical statements are. Overall, there were two common errors which were error in algebraic process and error in use definition correctly. *First*, errors in use definition correctly. five of thirteen students made error in using definition of even numbers. All of them knew the definition of even numbers was $2k$. Unfortunately, they did not explain detail about k. It should be a further explanation that k is any integers. If no description, k can be integers or real numbers or rational numbers or etc. It cannot be accepted since definition of even numbers is not clear. Another error about use of definition can be seen in Fig. 1. In Fig. 1, S4 made the form of even number by *2x*, it confused since based on the question variable used is *x* as even numbers. It's better if he used another letter as even number, like *2k* or *2m* where *k* or *m* are integers. Failed in using a definition correctly was one of the errors and as difficulty in solving general mathematical problems and also in constructing mathematical proof [10]. The error showed in Fig. 1 could be caused by several reasons. One of them was carelessness. A study showed that carelessness was one of the reasons of errors due when solving mathematics problems [14].

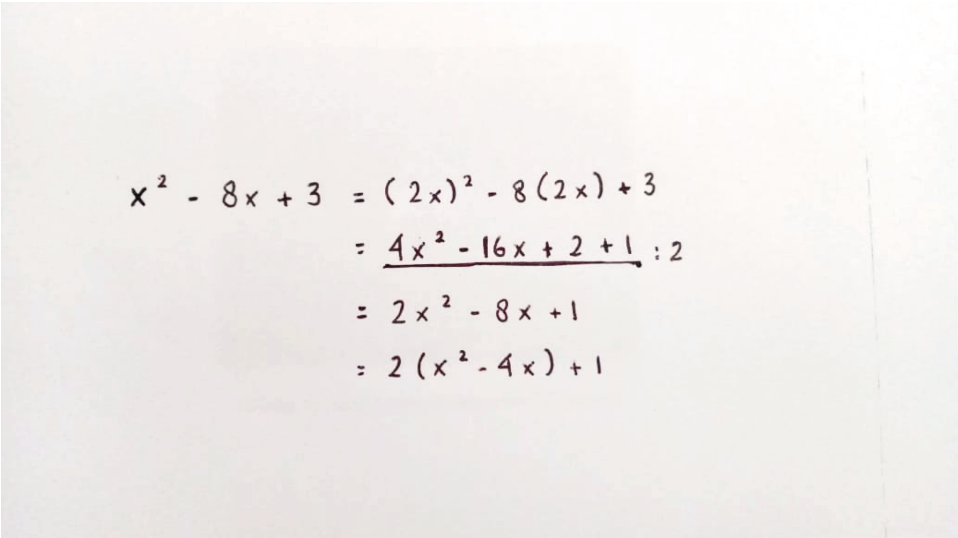


Figure 1: Error made by S4.

Second, errors in algebraic manipulation. Six of thirteen students made error in algebraic process when proving the statement in the question 1. The sample errors were shown in Fig. 1, Fig. 2, and Fig. 3. The error in Figure 1 was in second line. S4 divided all by 2, it can be seen in line 3 that 2 divided by 2 was equal to 0. Thus, S4 got wrong final answer. It occurred because lack of understanding about basic operation concept. It is in line with a research shown that lack of understanding about a concept would made error in solving a question [6]. The another research was also shown that lack of conceptual understanding was one of the source difficulties in constructing mathematical proof [10]. In Figure 2, It could be seen that S5 was right in supposing the form of even number, *2k*. But, in line 2, it showed that the part of algebraic process was wrong. Actually, the result of $(2k)^2 = 4k^2$. S5 wrote *4k* in line 2 and line 3. But the square was appeared in the last line. This is not consistent and it cannot be right. The similar answer was also presented in Fig. 3. There were two errors: (1) S11 did not put parentheses in line 2. It should parentheses such that $(2k)^2$. *2k* and $(2k)^2$ have a different meaning. The student did not understand how important parentheses really are; (2) In Line 2, S4 also made error, 3 expanded to *2k + 1*; (3) In line 3, S4 wrote 4k as the result of $2k^2$. It is obviously wrong. The errors presented above can be categorized procedural errors because the students did not follow the correct step when constructed the mathematical proofs. Beside procedural errors, in Figure 1, 2, and 3 also can be seen factual errors, an error when the students not mastered basic facts required in solving a mathematics problem. Factual and procedural error was also two of the common errors in solving mathematics problems [15].

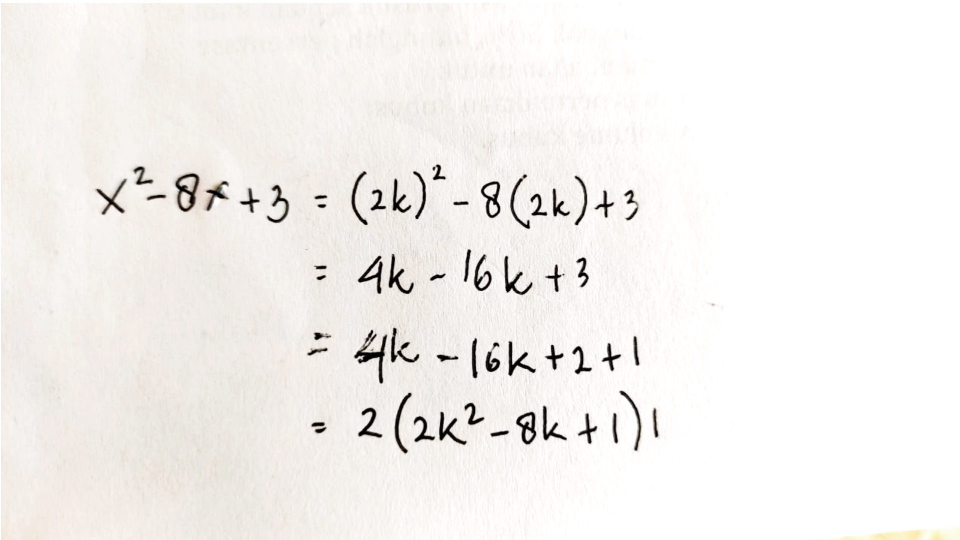


Figure 2: Error made by S5.

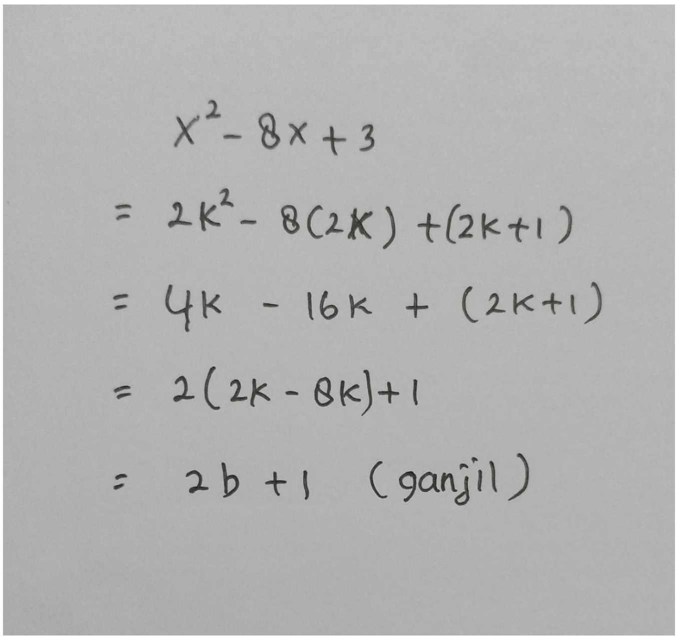
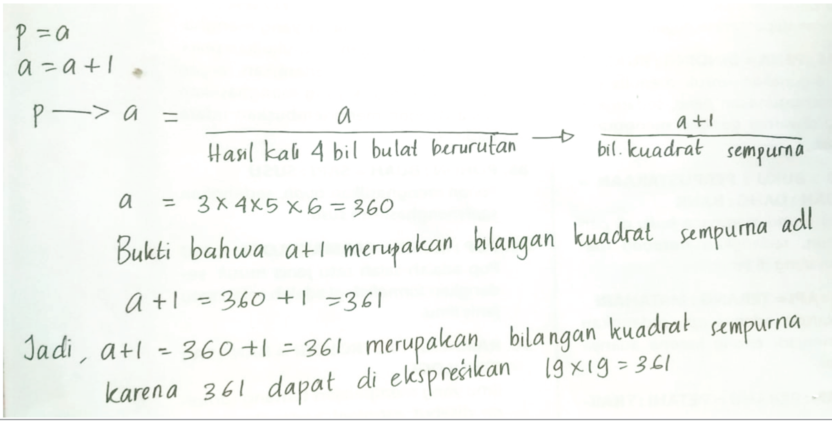


Figure 3: Error made by S11.

3.2. The Students' Errors in Question Number 2

The question in number 2 was “Suppose that *a* is the product of four consecutive integers. Prove that *a* + 1 is perfect square number”. In proving the statement, students have to construct four consecutive integers, it means they must know what the forms of four consecutive integers are. They also have to know the definition of perfect square numbers. Furthermore, the ability of algebraic manipulation is also needed. This ability is used to simplify a form into perfect square number form.

The analysis’ result of students answer indicated that this question is harder than question number 1. Only one student could prove correctly, the others failed to prove it. Twelve students prove it by example. This is unacceptable in mathematics. The result is in line with another research that revealed proof by example as one of the common errors in constructing mathematical proof [16]. The sample of answer is presented in Figure 4 and 5.



$p = a$

$a = a + 1$

$$p \to a = \frac{a}{product\ of\ 4\ consecutive\ integers} \to \frac{a+1}{perfect\ square\ number}$$

$a = 3 \text{ x } 4 \text{ x } 5 \text{ x } 6$

Proof that $a + 1$ is perfect square numbers is

$a + 1 = 360 + 1 = 361$

So, $a + 1 = 360 + 1 = 361$ is a perfect square number since 361 can be expressed as 19 x 19 = 361

Figure 4: Example of student answer (S9) in indonesian and english version.

The students in Figure 4 knew what the hypothesis and the conclusion are. S9 also understood the form of perfect square number. Unfortunately, S9 did not prove in general but proof by example. S9 gave four consecutive numbers i.e. 3, 4, 5, 6 then multiplied them (*a* = 360). For *a* + 1 = 360 + 1 = 361, it could be expressed as

DOI 10.18502/kss.v9i13.15969

19 x 19. Since 361 equals 19 x 19 then 361 is perfect square. Thus, a + 1 is a perfect square number such that the statement is proved. The procedure is right but it cannot be accepted as a mathematical proof. It is just an example to show the statement is hold. Proving by example in Fig. 4 showed that one of the students' difficulties in constructing mathematical proof was did not know how to start the proof. So, they were given example as the answer to avoid blank answer sheet. This support by Moore that included starting prove as one of the students' difficulties [10].

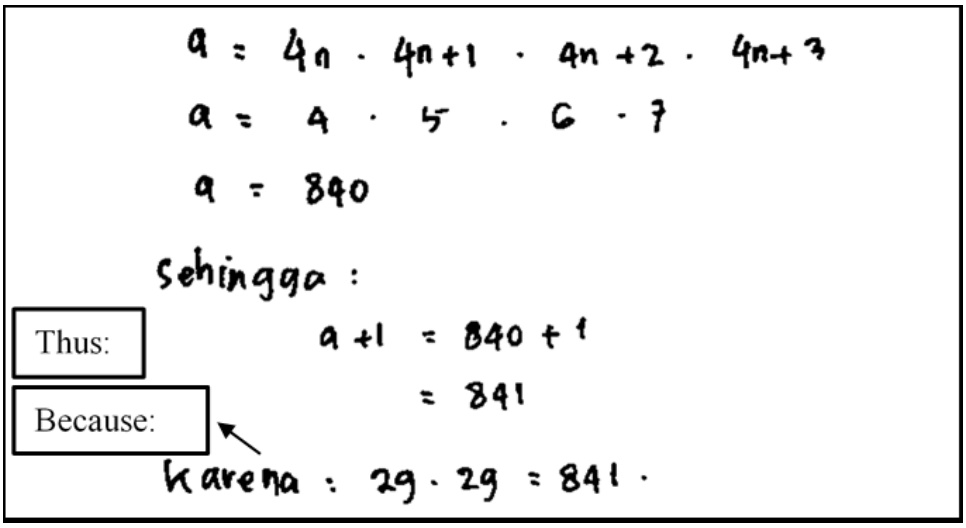


Figure 5: Example of student answer for question 2 (S3).

Another type of proving by example is presented in Figure 5. In the first line, S3 had written in general form of four consecutive integers. But, in line 2, S3 gave specific numbers (4,5,6,7) then multiplied all of them. The result was 840. When added by 1, it can be expressed as product of two equal numbers. The value of a + 1 = 840 + 1 = 841 = 29 x 29. Thus, it is perfect square number, so that it is the end of the proof. Like the explanation for Fig. 4, answer in Fig. 5 is also not a proof. The error in Figure 5 also indicates that students are still difficult in constructing mathematical proof. Overall, this study reveals that constructing mathematical proof by direct methods is still a problem for college students majoring in mathematics education.

4. CONCLUSION

Mathematical proving is a most important ability in mathematics. There are several methods for proving mathematical statements, one of them is direct methods. This method is often used in proving a statement. The common errors in proving mathematical statement by indirect method are error in using definition correctly, errors in algebraic process, and errors due to proving by example. By pinpointing the errors, lecturers can aware and give more attention in learning process to avoid the errors in the future. This condition is also hold for the students so that they will avoid to make the same error when constructing mathematical proofs. The further research is needed to create activities (teaching and learning model/strategy/methods) integrating mathematical proving process. This is imperative so as to avoid monotone in mathematics learning and as effort to improve mathematical proving ability of students of mathematics education study program (pre-service mathematics teachers).

Acknowledgments

We would like thank to all participants in both conducting the research and writing the article.

References

[1] Kemendikbud. "Peraturan menteri pendidikan nasional RI no. 22 tahun 2006. tentang standar isi kurikulum pendidikan dasar dan menengah." Departemen Pendidikan Nasional. p. 2006.

[2] Principles N. "Standards for school mathematics reston, va natl.," Counc. Teach. Math. p. 2000.

[3] Doruk M, Kaplan A. "Prospective mathematics teachers' difficulties in doing proofs and causes of their struggle with proofs," 1st International Eurasian Educational Research Congress. pp. 315–328, 2015.

[4] Rav Y. "Why do we prove theorems?" philosophia mathematica. vol. 7, no. 1, pp. 5–41, 1999. https://doi.org/10.1093/philmat/7.1.5.

[5] Hersh R. Proving is convincing and explaining. Educ Stud Math. 1993;24(4):389–99.

[6] Herizal H, Suhendra S, Nurlaelah E. The ability of senior high school students in comprehending mathematical proofs. Journal of Physics: Conference Series. Institute of Physics Publishing; 2019. https://doi.org/10.1088/1742-6596/1157/2/022123.

[7] Coe R, Ruthven K. Proof practices and constructs of advanced mathematics students. Br Educ Res J. 1994;20(1):41–53.

[8] Güler G, Dikici R. Examining prospective mathematics teachers' proof processes for algebraic concepts. Int J Math Educ Sci Technol. 2014;45(4):475–97.

[9] Weber K. Student difficulty in constructing proofs: the need for strategic knowledge. Educ Stud Math. 2001;48(1):101–19.

[10] Moore RC. Making the transition to formal proof. Educ Stud Math. 1994;27(3):249–66.

[11] Houston K. How to think like a mathematician: a companion to undergraduate mathematics. Cambridge: Cambridge University Press; 2009. https://doi.org/10.1017/CBO9780511808258.

[12] Mertens DM. Research and evaluation in education and psychology: integrating diversity with quantitative, qualitative, and mixed methods. Sage publications; 2019.

[13] Creswell JW. Research design: qualitative, quantitative, and mixed methods approaches. California: SAGE Publications, Inc; 2014.

[14] Tong DH, Loc NP. Students' errors in solving mathematical word problems and their ability in identifying errors in wrong solutions. Eur J Educ Stud. 2017;3(6):226–41.

[15] Fei Lai C. Technical report: error analysis in mathematics. Eugene; 2012.

[16] Stavrou SG. Common error and misconceptions in mathematical proving by education undergraduates. Issues in the Undergraduate Mathematics Preparation of School Teachers. 2014;1:1–8.